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Neutron rich Carbon and Oxygen isotopes with an odd number of neutrons

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Abstract. We describe odd isotopes as formed of a core plus one neutron. We have calculated the modification of single neutron energies brought by couplings of the neutron with collective 2^+ phonons in the cores. The results reproduce very well the inversion of $2s$ and $1d_{5/2}$ shells in carbon isotopes up to ^{19}C while in oxygen isotopes the correction is also large but do not show any inversion in agreement with experiments. The calculated energies are close to the experimental ones in both series of isotopes except in ^{21}C for the $2s$ state for which our coupling is too weak.

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1 Introduction

In the last decade the properties of carbon and oxygen isotopes have been extensively studied experimentally and theoretically. They show very interesting new behaviour when one compares with heavier nuclei. In particular carbon isotopes offer a very interesting situation. Indeed in lighter carbons, including ^{12}C, one notices an inversion of the $2s$ and $1d_{5/2}$ shells, inversion which disappears when N, the number of neutrons, is equal to 12 or 14. At contrary, in oxygen isotopes the normal sequence of $1d_{5/2}$ and $2s$ shells with the $1d_{5/2}$ lower than the $2s$ one is observed but neutron energies for states closed to the Fermi surface are far from energies given by a Saxon Woods potential or an Hartree Fock potential [1]. Therefore we are interested to see what is the origin of this inversion in carbon nuclei and how the situation is different in oxygen nuclei.

These nuclei have been the object of calculations mostly using shell models but we present a different description based on the assumption of a two-body system formed of a neutron added to an inert core for which the two-body correlations are introduced via a coupling between the neutron and a phonon of the core. The corrections to the one-body neutron-core potential [2] due to this type of coupling induces a modification of the single neutron energies which can be easily calculated from the known properties of the phonons of the core. In section 2 we present our model and detail the calculation of corrections due to two-body correlations introduced via neutron-phonon couplings. Then in section 3 we present and discuss the corresponding results obtained for carbon and oxygen isotopes. At last section 4 is devoted to our conclusion.

2 Description of isotopes with an odd number of neutrons and contribution of Neutron-phonon couplings to the single neutron energies

Assuming a neutron plus an inert core, the neutron states will be calculated first using a Saxon-Woods one-body potential for the neutron-core interaction written as [3]:

$$V(r) = -V_0 \left(f(r) - 0.44 r_0^2 (\mathbf{l}.\mathbf{s}) \frac{1}{r} \frac{df(r)}{dr} \right) \qquad (1)$$

where

$$f(r) = \left[1 + exp\left(\frac{r - R_0}{a} \right) \right]^{-1} \text{ and} \qquad (2)$$

$$V_0 = \left(50.5 - 32.5 \, \frac{N-Z}{A} \right) \, MeV \qquad (3)$$

with a parameter a of 0.75 fm, larger than usual, to take account of the diffuse surface of such light nuclei. The other parameters are $R_0 = r_0 A^{1/3}$ with r_0=1.27 fm where A is the nucleon number in the core. These parameters were fitted in order to reproduce at best the neutron shells in nuclei with $A > 40$ where particle-phonon couplings are expected to have little effect on single particle energies, therefore it can be considered as a good phenomenological representation of the first order one-body potential.

To this first order potential we add the contribution due to two-body correlations taken into account via a coupling between the neutron and a phonon of the core. The corresponding modification of the potential is given in references [2,4,5]. To calculate the corresponding modification to the single neutron energies we follow reference [2].

This correction, $\Delta\epsilon_n$ for the neutron in state n is given by:

$$\Delta\epsilon_n = \Sigma_{N,\lambda} F_{N,\lambda} \left| \int \phi_n^*(r) V_{0N}(r) \phi_\lambda(r) dr \right|^2 \qquad (4)$$

where:

- N and λ mean respectively the phonons which are included in the calculation and the state of the intermediate neutron.

- $\phi_n(\mathbf{r})$ and $\phi_\lambda(\mathbf{r})$ are respectively the wave functions calculated in the potential of Eq. 1 for the neutron in states n and λ with respective energies ϵ_n and ϵ_λ.

- V_{0N} is the amplitude for exciting the phonon N of angular momentum L. As in reference [2] we make a phenomenological approach and write V_{0N} as:

$$V_{0N}(\mathbf{r}) = \frac{1}{(2L+1)^{1/2}} \beta_L R_0 \frac{dU(r)}{dr} Y_L^{M*}(\omega_r) \qquad (5)$$

where U(r) is the central part of our Saxon Woods potential of Eqs. (1-3) and β_L is related to the B(EL) by:

$$B(EL) = \left[\frac{3Z}{4\pi} R_c^2 \beta_L \right]^2 e^2 fm^4 \qquad (6)$$

- $F_{N,\lambda}$ is a coefficient given by:

$$F_{N,\lambda} = \left(\frac{1-n_\lambda}{E_n - \epsilon_\lambda - \omega_N}\right) + \left(\frac{n_\lambda}{E_n - \epsilon_\lambda + \omega_N}\right) \qquad (7)$$

where n_λ is the occupation number of the state λ and E_n is the energy of the studied neutron after correction, which is then given by:

$$E_n = \epsilon_n + \Delta\epsilon_n \qquad (8)$$

These equations have been solved by iteration to get the new energy E_n.

3 Results for carbon and oxygen isotopes

3.1 Carbon isotopes

All the results are given in table 1 for the states closed to the Fermi surface. We first concentrate on the energies of the $2s$ and $1d_{5/2}$ shells which present an unusual behaviour. We first calculate their energies in our Saxon Woods potential of Eq. 1. We see that up to ^{20}C the $2s$ state is lower than the $1d_{5/2}$, then the order of the two shells is inverted and the usual situation is recovered for ^{20}C. The same situation was already observed for ^{11}Be in [2] when a pure Saxon Woods potential was used. This property is clearly related to the radius of the neutron-core potential. Indeed for smaller radii the centrifugal barrier is more efficient and pushes up the d-states compared to the $2s$ one. However the calculated energies are quite far from the experimental ones [6,7,8,9,10,11,12,13,14,15,16,17, 18,19,20,21,22,23] which are also given in the table 2.

Table 1. Results for carbon isotopes and comparison with experimental values when they are known.

		$2s_{1/2}$	$1d_{5/2}$	$1d_{3/2}$	$2d_{5/2}$	$2d_{3/2}$
^{12}C	ϵ_n	−1.02	−0.76	1.80	1.94	3.83
	$\Delta\epsilon_n$	−0.93	−0.38	−0.04	−0.08	−0.67
	E_n	−1.95	−1.14	1.76	1.86	3.16
	E_{exp}	−1.86	−1.12		1.74	3.3
^{14}C	ϵ_n	−0.93	−0.73	1.76	1.93	3.6
	$\Delta\epsilon_n$	−0.21	−0.06	−0.01	−0.01	−0.16
	E_n	−1.13	−0.79	1.75	1.92	3.44
	E_{exp}	−1.22	−0.48			3.47
^{16}C	ϵ_n	−0.90	−0.78	1.72	1.92	3.38
	$\Delta\epsilon_n$	−0.37	0.	−0.03	0.03	−0.11
	E_n	−1.27	−0.78	1.69	1.95	3.27
	E_{exp}	−0.73	−0.48			3.2
^{20}C	ϵ_n	−0.93	−1.02	1.63	1.89	2.91
	$\Delta\epsilon_n$	0.37	−0.18	0.05	0.01	−0.02
	E_n	−0.56	−1.20	1.68	1.90	2.89
	E_{exp}	0.5				3.

Table 2. Energies and $B(E2)$ of the 2^+ states used in our calculation for the carbon cores

	^{12}C	^{14}C	^{16}C	^{20}C
E2 (MeV)	4.43	7.01	1.76	1.62
B(E2) (e^2fm^2)	39.7	18.7	13.	18.4

Therefore we expect two body correlations to be important as it was the case for the inversion of $1p_{1/2}$ and $2s$ shells in ^{11}Be, ^{10}Li and ^{13}Be. Indeed the cores that we are considering have 2^+ states with a low excitation energy and very large $B(E2)$ due to a large collectivity. Such collective 2^+ states were shown to be responsible for a large modification of neutron energies in a core of ^{10}Be and to give most of the contribution to the corrective term [2, 24]. We give in Table II the energies and B(E2) for the 2^+ excited states that we have included in our calculation [25], assuming that it gives most of the contribution. The results for cores of ^{12}C, ^{14}C, ^{16}C and ^{20}C are given in the table 1 where we see that now the inversion of the two shells is amplified and individual neutron energies for $2s$ and $1d_{5/2}$ shells are close to the experimental ones. For a ^{18}C-core we did not calculate the corrective term. Indeed from the calculation in a SW potential and as given by experimental energies the two shells are very closed, so that ^{18}C is very likely a mixture of $(2s)^2$ and $(1d_{5/2})^2$ configurations for the two last neutrons. It is indeed verified when it is described as a core of ^{16}C plus two neutrons [26]. Therefore in our model it may not be taken as a core.

The same calculation has been done for $1d_{3/2}$, $2d_{5/2}$ and $2d_{3/2}$ shells and results are given in table 1. For higher neutron states the corrections due to the couplings are negligible as expected. Our $2d_{5/2}$ reproduces very well the second $5/2^+$ excited state in ^{13}C [15] while there is no experimental $3/2^+$ state corresponding to our $1d_{3/2}$ state. This state is very close to the second $5/2^+$ and perhaps not easily detected experimentally. However our $2d_{3/2}$ is

Table 3. Results for oxygen isotopes and comparison with experimental values when they are known [28,30,34].

		$2s_{1/2}$	$1d_{5/2}$	$1d_{3/2}$
^{14}O	ϵ_n	−3.5	−4.92	1.53
	$\Delta\epsilon_n$	−0.89	−0.79	−0.27
	E_n	−3.90	−5.02	0.99
	E_{exp}	−4.50	−6.4	1.54
^{16}O	ϵ_n	−3.2	−4.52	1.37
	$\Delta\epsilon_n$	−0.61	−0.54	−0.5
	E_n	−3.81	−4.05	0.87
	E_{exp}	−3.3	−4.14	0.94
^{22}O	ϵ_n	−2.85	−4.1	0.65
	$\Delta\epsilon_n$	0.12	0.15	0.26
	E_n	−2.73	−3.95	0.91
	E_{exp}	−2.74	−3.8	
^{24}O	ϵ_n	−2.82	−4.08	0.38
	$\Delta\epsilon_n$			
	E_n			
	E_{exp}	−4.09	−5.05	0.77

in great agreement with the known high $3/2^+$ state in all our systems even though the corrective term is very small.

For a ^{20}C core we recover the usual order with the $1d_{5/2}$ lower than the $2s$, therefore ^{20}C may be assumed to have a closed $1d_{5/2}$ shell. However for the $2s$ state the neutron-phonon coupling is not strong enough to make the state unbound as it should. In this case we may expect a further contribution to the potential of a term corresponding to the excitation of collective pairing vibrations in the $N-2$ and $N+2$ systems [4], therefore in ^{18}C and ^{22}C. Indeed in particular in ^{18}C it has been shown that the wave functions are mixture of several configurations for the two last neutrons [26]. This type of contribution was found long ago to have a noticeable contribution in ^{40}Ca [27].

As a summary we can say that, even though we have restricted the neutron-phonon coupling to the low energy 2^+ phonon, we have been able to reproduce the inversion of $2s$-$1d_{5/2}$ shells in the lightest carbon isotopes (up to ^{20}C) and could get reasonable agreement with experimental neutron energies for all calculated states.

3.2 Oxygen isotopes

We have performed the same calculations for oxygen isotopes assumed to be described as an inert core plus one neutron. We have considered cores of ^{14}O, ^{16}O, ^{20}O and ^{25}O.^{18}O may not be assumed as a core. Indeed because of the proximity of the $2s$ and $1d_{5/2}$ shells, the wave function is a mixture of two configurations with the two last neutrons in one of these two shells. As for carbons we have calculated the contribution to the neutron energies of neutron-phonon couplings due to the low energy 2^+ excited state of the core. The energies and B(E2) are taken from measurements [28,29] and given in table 4. The results of our calculation are reported in table 3 for $2s$, $1d_{5/2}$ and $1d_{3/2}$ neutron states together with the experimental energies [28,30,31,32,33].

Table 4. Energies and $B(E2)$ of the 2^+ states in the oxygen cores used in our calculation.

	^{14}O	^{16}O	^{22}O
E2 (MeV)	6.59	6.92	3.19
B(E2) (e^2fm^2)	43.	40.6	21.

We see first that we have no inversion of the $2s$ and $1d_{5/2}$ shells for any of the isotopes in agreement with experiments. On the other side the neutron-phonon couplings are quite important and put the calculated energies close to the experimental ones. As in carbon isotopes, for higher states and already for the $2d_{5/2}$ and $2d_{3/2}$ states, these corrections are negligible (<0.1 MeV). For ^{25}O we do not know any 2^+ state then could not calculate the coupling term. However we note that to obtain the experimental energy, in particular for the $2s$ state, we need a large corrective term what suggests that there is a low 2^+ state in ^{24}O with a large B(E2). This is plausible since in ^{24}O the $1d_{5/2}$ and $2s$ shells are closed to each other and a 2^+ state can be constructed by coupling two neutrons of the $1d_{5/2}$ shell to $J^\pi = 2^+$. The excitation energy of this state should be then close to the low 2^+ state of ^{18}O known at 1.98 MeV.

4 Conclusions

To conclude we were able to understand and reproduce the properties of single neutron states in carbon and oxygen isotopes with an odd number of neutrons. We have shown the importance of two body correlations which are treated in our work as neutron-phonon couplings reduced to coupling with the lowest 2^+ state in the core. In particular we could explain that for carbon isotopes the $2s$ and $1d_{5/2}$ shells are inverted up to ^{20}C as shown experimentally. For oxygen isotopes no such inversion appears in our calculations as required by experiments. For both series of nuclei we get neutron energies closed to experimental values except for ^{21}C where the coupling of the neutron with the 2^+ phonon of ^{20}C is not strong enough to get the measured unbound energy of the $2s$ state even though it reduces the discrepancy between calculation and measurement.

Apart the corrections on single neutron energies, particle-phonon couplings induce a modification of the wave functions which are now a mixture of one single state with configurations where one neutron is coupled to phonons [2]. One, sometimes, makes a parallel between this mixture of complex configurations and a deformation of the corresponding state as it is explicitly used in sulfur studies [35]. This equivalence was already clear in the case of ^{11}Be where the inversion of $1/2^+$ and $1/2^-$ was shown to be due to two-body correlations in a calculation similar to the present one [2] or to a deformation of the one-body average potential [36,37] due to the presence of the low 2^+ state in ^{10}Be which was considered as a vibrational state in the first case and a rotational state in the second one.

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